Parallel Push-Pull Hybrid Circuit*

MASAHISA MIYAGI†, SENIOR MEMBER, IRE

Summary—This paper deals with a new hybrid circuit for signal branching or combining in parallel push-pull amplifiers.[1] The purpose of the new hybrid circuit is to perform the branching or combining of two pairs of signals for parallel push-pull operation, each consisting of two oppositely phased, balanced signals. The advantage of combining two pairs of parallel push-pull signals by a single component of new hybrid circuit is clear; one would require a multiplicity of conventional rat-race circuits to do the same job, inasmuch as the basic rat-race circuit is limited to only one pair of push-pull signals.

The new hybrid circuit is further elaborated to improve the performance for combining unbalanced push-pull signals by: (1) showing that the combination of unbalanced push-pull signals is equivalent to the super-position of cophase components on the balanced antiphase components; (2) analyzing the effect of the cophase components in the hybrid circuit; and (3) devising means of improving the adverse effects of the cophase components.

Finally, an evaluation is made of a parallel push-pull amplifier using the new hybrid circuit in comparison with the conventional rat-race circuits required for the same parallel push-pull operation.

I. Introduction

THE PROPOSED parallel push-pull hybrid circuit is used for the purpose of branching simultaneously two pairs of signals, equal in amplitude and opposite in phase—that is, parallel push-pull signals, or conversely, combining two pairs of signals simultaneously. The conventional circuits, such as the so-called rat-race circuits, can handle only one pair of push-pull signals. When two pairs of signals are to be handled, a total of three rat-race circuits must be used by connecting in two-stage cascade. In a like manner, the input and the output of a parallel push-pull power amplifier would require a total of six rat-race circuits. This not only makes the construction of such an amplifier circuit complex and maintenance tedious, but also results in poor performance.

In view of the ever-increasing demands for larger power-handling capacities of the microwave over-the-horizon multichannel systems and the restrictions imposed on power-handling capabilities of transmitting tubes, the author made an effort to develop a simpler hybrid circuit capable of branching out or combining parallel push-pull signals. Although an analysis is made here of the behavior of the parallel push-pull hybrid circuit for a case of two pairs of signals, it should be understood that the hybrid circuit also has application in branching out or combining multipair parallel push-pull signals.

The features of the new parallel push-pull hybrid circuit are summarized below:

1) The hybrid circuit is of single-stage construction—that is, extremely simple in construction. This feature is particularly advantageous in handling multipair parallel push-pull signals.
2) The circuit is designed to have equal input or output termination impedances.
3) A change in output level resulting from a change in impedance or signal level is smaller than that for the rat-race hybrid circuit. This effect will be analyzed later in this paper by dividing the input into parallel push-pull signals, recombining them into a single output signal, and comparing the output signal level stability between two cases, one for a rat-race branching and combining circuit, and the other for a hybrid branching and combining circuit.

In the description and analysis of the new hybrid circuit that follows, signal combination and branching will be considered separately for cophase and antiphase components, for ease of understanding.[2]

Fig. 1 shows an example of the rat-race hybrid circuit construction for use in parallel push-pull signal combination and branching.[3]

Referring to Fig. 1, let Y_0 be the characteristic admittance of either the input or the output terminal. Then the characteristic admittance of the ring circuit Y_1 is expressed as

$$Y_1 = \frac{Y_0}{\sqrt{2}}.$$

In Fig. 1, terminals ① and ③ are cophase and so are terminals ② and ④, but between the two terminal groups exists a phase difference of 180°. Terminal ⑤ is used as an input terminal in branching and as an output terminal in combining.

In the parallel push-pull circuit, six parallel branching terminals are provided along a ring circuit with characteristic admittance Y_0, and these six terminals are spaced apart in succession at distances $\lambda_0/4$, $\lambda_0/2$, $\lambda_0/4$, $\frac{3}{4}\lambda_0$, $\lambda_0/2$, and $\lambda_0/4$, each terminal being terminated with characteristic admittance Y_0.

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† Nippon Electric Co. Ltd., Kawasaki, Japan.

[1] M. Miyagi, "Parallel push-pull hybrid circuit," *J. Inst. Elec. Commun. Engrs. Japan*, vol. 43, pp. 52–58; October, 1960.

[2] J. Reed and G. J. Wheeler, "A method of analysis of symmetrical four-port networks," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 246–252; October, 1956.

[3] W. A. Tyrell, "Hybrid circuits for microwaves," Proc. IRE, vol. 35, pp. 1294–1306; November, 1947.

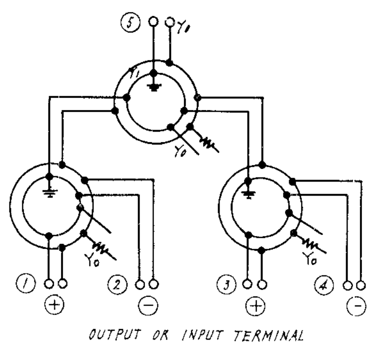


Fig. 1—An example of a branching or combining circuit using three "rat-race" circuits.

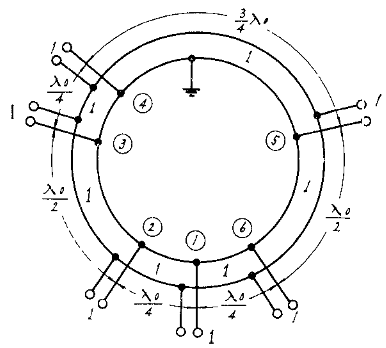


Fig. 2—Normalized parallel push-pull hybrid circuit.

Fig. 2 shows the hybrid circuit normalized by the characteristic admittance using the center frequency as a parameter, in which various factors due to electrical discontinuties at these terminals are disregarded.

The same parallel terminal numbers as in Fig. 2 will be used throughout the analysis to follow.

II. Parallel Push-Pull Branching

Let us analyze a case in which a normalized signal of unity is applied to terminal ① in Fig. 2. The construction of the hybrid circuit will be symmetrical about a vertical center axis, except that there is a difference in electrical length of $\lambda_0/2$ between terminals ③–④ and terminals ④–⑤.

Now let us consider the components $+\frac{1}{2}$, $+\frac{1}{2}$ applied to terminal ①, $+\frac{1}{2}$, $-\frac{1}{2}$ applied to terminal ④ in lieu of the normalized unity signal. Then these components may be considered as equivalent to two pairs of signals $+\frac{1}{2}$ and $+\frac{1}{2}$ respectively applied to terminals ① and ④ (cophase components) and $+\frac{1}{2}$ and $-\frac{1}{2}$ respectively applied to terminals ① and ④ (antiphase components). After computing each signal component behavior individually and applying the superposition theorem, the circuit performance can be analyzed for a case of applying the unity signal wave to terminal ①.

A. Cophase Component Performance

Fig. 3(a)–(d) shows a set of equivalent circuits for various cases. Two branch circuits, including terminal ② and ③ and the other terminals ⑤ and ⑥, are connected across terminals ① and ④. Since this circuit is symmetrical about a vertical center axis, except that there is a difference in length of $\lambda_0/2$ between ③–④ and ⑤–④, the voltage maximum will appear at the symmetry point A-A', or at a point removed $\lambda_0/4$ from terminals ② and ③ for cophase components.

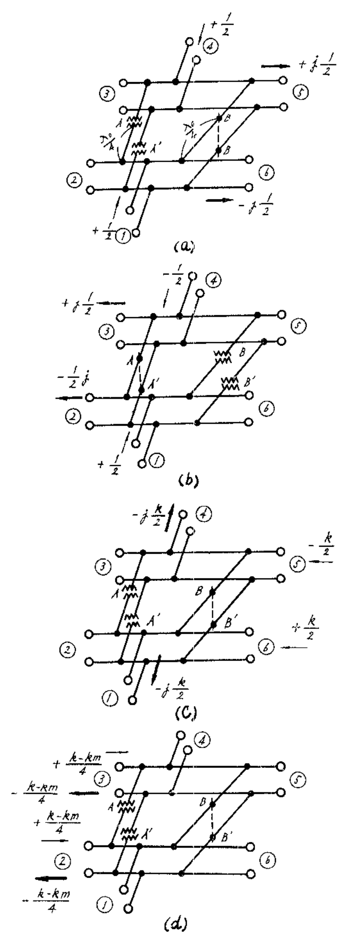


Fig. 3—Equivalent circuits. (a) Cophase branching. (b) Antiphase branching. (c) Single balanced, antiphase pair combining. (d) Single balanced, cophase pair combining.

Therefore, the voltage distribution between terminals ② and ③ will become identical with that which would be obtained if the line is opened at the points A-A'.

The signal traveling from terminal ④ to ⑤ will be phase-inverted at a point removed $\lambda_0/2$ from terminal ④. Therefore, the voltage distribution between terminals ④ and ⑤ produced by the signal will become identical with that which would be obtained if the antiphase components were applied to the hypothetical symmetrical hybrid circuit (or one modified so that the length between terminals ④ and ⑤ equals $\lambda_0/4$).

Therefore, the voltage minimum will appear at the symmetry point B-B'. In other words, the voltage distribution becomes the same as that which would be obtained if the line were short-circuited at points B-B'.

Let the transfer and reflection coefficients be expressed by t and γ, respectively, and let the subscript denote transfer or reflection paths between terminals, *i.e.*, let t_{16} denote transfer from ① to ⑥, and γ_{11} denote reflection at ①.

Then the following relations will be established (note: $\frac{1}{2}$ denotes that the amplitude of each signal wave is $\frac{1}{2}$):

$$\begin{aligned} \tfrac{1}{2}\gamma_{11} &= 0, & \tfrac{1}{2}t_{16} &= -\tfrac{1}{2}j, & \tfrac{1}{2}t_{12} &= 0 \\ \tfrac{1}{2}\gamma_{44} &= 0, & \tfrac{1}{2}t_{45} &= +\tfrac{1}{2}j, & \tfrac{1}{2}t_{43} &= 0. \end{aligned} \tag{1}$$

B. Antiphase Component Performance

As shown in Fig. 3(b), let us consider a case in which the antiphase components are applied to terminals ① and ④. Then the over-all voltage distribution will become identical with that which would be obtained if the line were short-circuited at point A-A' and open-circuited at points B-B'.

Then the following relations are established:

$$\begin{aligned} \tfrac{1}{2}\gamma_{11}' &= 0, & \tfrac{1}{2}t_{16}' &= 0, & \tfrac{1}{2}t_{12}' &= -\tfrac{1}{2}j \\ \tfrac{1}{2}\gamma_{44}' &= 0, & \tfrac{1}{2}t_{45}' &= 0, & \tfrac{1}{2}t_{43}' &= -\tfrac{1}{2}j. \end{aligned} \tag{2}$$

(Note: The prime is to distinguish antiphase from cophase.)

C. Combined Branching Performance

The over-all operation of this hybrid circuit when the unity normalized input signal is applied to terminal ① can be computed by applying the superposition theorem to the cophase and antiphase components. In other words, the inputs applied to terminal ④ cancel each other and vanish.

As a result, +1 signal is applied to terminal ① as the input signal. There will be no signal transfer or reflection between any two terminals except for transfer or reflection of the components as indicated in Fig. 3 (a) and (b).

Let the over-all reflection and transfer signals be expressed respectively by Γ and T. Then from (1) and (2), the following equations are derived:

$$\left.\begin{aligned} \Gamma_{11} &= \tfrac{1}{2}\gamma_{11} + \tfrac{1}{2}\gamma_{11}' = 0 \\ T_{12} &= \tfrac{1}{2}t_{12} + \tfrac{1}{2}t_{12}' = -\tfrac{1}{2}j \\ T_{43} &= \tfrac{1}{2}t_{43} - \tfrac{1}{2}t_{43}' = +\tfrac{1}{2}j \\ \Gamma_{44} &= \tfrac{1}{2}\gamma_{44} - \tfrac{1}{2}\gamma_{44}' = 0 \\ T_{45} &= \tfrac{1}{2}t_{45} - \tfrac{1}{2}t_{45}' = +\tfrac{1}{2}j \\ T_{16} &= \tfrac{1}{2}t_{16} + \tfrac{1}{2}t_{16}' = -\tfrac{1}{2}j \end{aligned}\right\}. \tag{3}$$

Care must be taken in deriving (3) whether the sign of $\frac{1}{2}$ signal applied to each terminal is plus or minus.

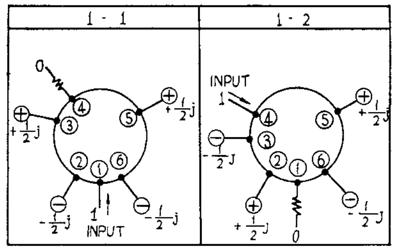
$$|\Gamma_{11}|^2 + |T_{12}^2| + |T_{43}^2| + |\Gamma_{44}^2| + |T_{45}^2| + |T_{45}^2| + |T_{16}^2| = 1$$

$$T_{12} = T_{16} = -T_{43} = -T_{45}. \tag{4}$$

Thus, the parallel push-pull signals equally divided into four parts are available from terminals ②, ⑥, ③, and ⑤ and there will be be perfect isolation between terminals ① and ④.

Table I summarizes these relations in schematic diagram form.

TABLE I
PARALLEL PUSH-PULL BRANCHING



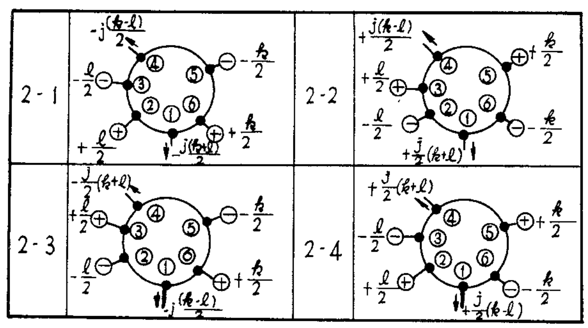
III. PARALLEL PUSH-PULL COMBINING

The operation of the circuit of Fig. 2 as a combining circuit will now be considered.

A. Single Balanced Antiphase Pair

Fig. 3(c) is an equivalent circuit of the single balanced antiphase pair combining circuit for a case in which a pair of input push-pull signals, equal in amplitude ($k/2$) and opposite in phase, is applied to terminal ⑤ and ⑥. To analyze the circuit operation, it is only necessary to put $l=0$ in Table II (2-1).

TABLE II
PARALLEL PUSH-PULL COMBINING CIRCUIT



In either case, the isolation between terminals ②-③ and ⑤-⑥ becomes infinite, this fact being extremely convenient in using this circuit as a combining circuit.

B. Two Balanced Antiphase Signal Pairs

Where two balanced push-pull signal pairs are applied respectively across terminals ②-③ and ⑤-⑥, the output signal can be computed by using the superposition theorem. The combined output becomes exactly equal to the vector sum of the input signal applied to each terminal, and each terminal impedance is matched to the circuit impedance.

For these reasons, it will be evident that the parallel push-pull circuit of Fig. 2 meets all conditions necessary for a parallel push-pull combining circuit.

C. Two Unbalanced Antiphase Push-Pull Signal Pairs

In the presence of unbalance between one pair of push-pull signals and the other, which are to be applied to the combining circuit, the unbalanced signal components will be absorbed in the termination resistance.

Thus the signal input and output impedances are unaffected even if the antiphase components are unbalanced, and hence, any one of these terminal impedances will be matched to the load.

IV. Considerations for Cophase Combining

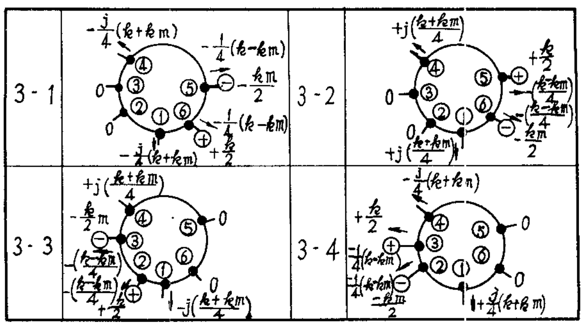
The presence of unbalance between a pair of push-pull signals indicates that cophase components exist, as well as antiphase components. This case will now be investigated.

A. Unbalanced Pair

Referring to 3-3 in Table III, analysis will be made of a case in which unbalance exists between two push-pull signals constituting a pair, one being $+k/2$ and the other $-(k/2)m$.

TABLE III

Combining Circuits Containing Cophase Components



It is assumed here that $0 \leq m \leq 1$. Let $+k/2$ and $-(k/2)m$ be considered as broken down into the cophase components $+k-km/4$, $+k-km/4$ and the antiphase components $+(k+km)/4$, $-(k+km)/4$.

Then these relations are given by

$$\left.\begin{aligned} +\frac{k}{2} &= \frac{k-km}{4}+\frac{k+km}{4} \\ -\frac{k}{2} &= \frac{k-km}{4}-\frac{k+km}{4} \end{aligned}\right\}. \qquad (5)$$

The behavior of the circuit as to the antiphase components can be easily analyzed from what has been described in Section III-A.

Fig. 3(d) illustrates an equivalent circuit for the cophase components. As is illustrated, all of the cophase components contained in the input signals are reflected.

In the case of 3-3 in Table III, there will be a reflected wave with an amplitude $k-km/4$ at each of the terminals ② and ③, with the result that only the antiphase components will appear at ① and ④ as the outputs.

In the foregoing analysis, the $(-)$ side amplitude is taken smaller than the $(+)$ side amplitude $(0 \leq m \leq 1)$. If otherwise, the phase of a reflected wave is simply reversed.

Let the equivalent voltage standing-wave ratio at any terminal that gives rise to reflections, for instance at terminal ⑥ in the case of 3-1 in Table III, be expressed by S.

Then S is given by

$$S = \frac{3-m}{1+m} \cdot \qquad (6)$$

The absolute value of the equivalent reflection coefficients at terminal ⑤ is $|(1-m)/2m|$.

B. Combining Balanced Push-Pull Pair and Unbalanced Push-pull Pair

Now let it be required to analyze the behavior of the combining circuit shown in 3-2 in Table III when a balanced pair $+k/2$, $-k/2$ and an unbalanced pair $+k/2$, $-(k/2)m$ are respectively applied to terminals ②–③ and ⑤–⑥. In this case, a resultant output with an amplitude $-jk(1-m)/4$ at terminal ① and an amplitude $j[k(3+m)/4]$ at terminal ④ is available, while there will be a reflected wave with an amplitude $-k(1-m)/4$ at each of terminals ⑤ and ⑥.

The behavior of the circuit can be analyzed in like manner by combining Table II with Table III when an unbalanced pair is applied to another pair of terminals.

C. Improved Combining Circuit

In the presence of cophase components, reflections at terminals to which unbalanced pairs are applied are unavoidable. A set of equivalent circuits of an improved combining circuit is shown in Fig. 4, which is capable of providing a satisfactory combining operation despite the presence of cophase components.

Each of the equivalent circuits shown in Fig. 4 differs in construction from the circuit of Fig. 2 in that two parallel paths are connected across terminals ② and ③, and further, a termination resistance $2Y_0$ is connected across the mid-points of the parallel paths, the same situation holding true for terminals ⑤ and ⑥.

Since the cophase components are absorbed in the newly installed termination resistances, the combining circuit provides normal operation despite the presence of cophase components in the input signals. For the cophase components, as shown in Fig. 4(a), the voltage distribution will become identical with that which would be obtained if the line were short-circuited at symmetry points A_1-A_1' and A_2-A_2' and opened at B_1-B_1'.

Thus the cophase components are absorbed in the termination resistance at B_2-B_2'. Further, as shown in Fig. 4(b), both termination resistances become unrelated to the antiphase components, which is sufficient to obtain the resultant output with respect to the antiphase components.

V. Application of the New Hybrid Circuit to a Power Amplifier

Evaluation of the performance of the new hybrid circuit is made by the author as applied to an input branching and to an output combining parallel push-pull amplifier circuit consisting of four vacuum tubes.

An analysis of this method is made below for the purpose of comparing the merits and demerits between the parallel push-pull circuit consisting of six rat-race circuits, and that consisting of two hybrid circuits.

1) An input signal is branched out to four signals for parallel push-pull excitation.
2) The four branched-out signals are recombined into an output after passing through respective unity-gain buffer amplifiers, inserted before the recombining hybrid circuit.
3) In this branching and combining circuit, one of the four terminal connections is decoupled from the hybrid circuit, and changes in stability of the operation of the hybrid circuit are evaluated before and after decoupling.

A. Rat-Race Circuits Performance

A computation is made of the over-all performance for a case in which terminals ①, ②, ③ and ④ of the rat-race branching circuit shown in Fig. 1 are connected to the corresponding terminals of another rat-race combining circuit of the same construction, and then, connection of terminal ① is decoupled. A total of six rat-race circuits, three for branching and three for combining, are used.

1) Input Branching Circuit: The table in Fig. 5(a) shows the over-all operation of the circuit when connection of terminal ① is opened. The VSWR at input terminal ⑤ is 1.66̇.

2) Output Combining Circuit: When the output signal shown in the table of Fig. 5(a) is applied to terminal ②, ③ and ④ in Fig. 5(b) and connection of terminal ① is opened, the resultant output power falls off to 56 per cent of the normal value and the VSWR of the output impedance decreases to 1.66̇. These relations are the same no matter which terminal connection is opened.

3) Input Impedance Characteristics: Curve A in Fig. 7 shows the input impedance response of a single rat-race circuit obtained by measurements, which coincides

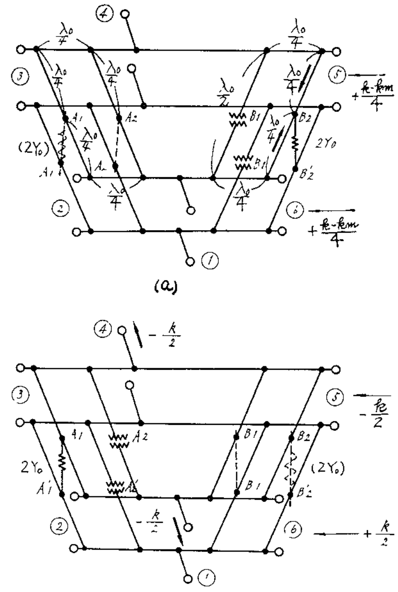


Fig. 4—Combining circuit considering cophase components. (a) Equivalent circuit for cophase components. (b) Equivalent circuit for antiphase components.

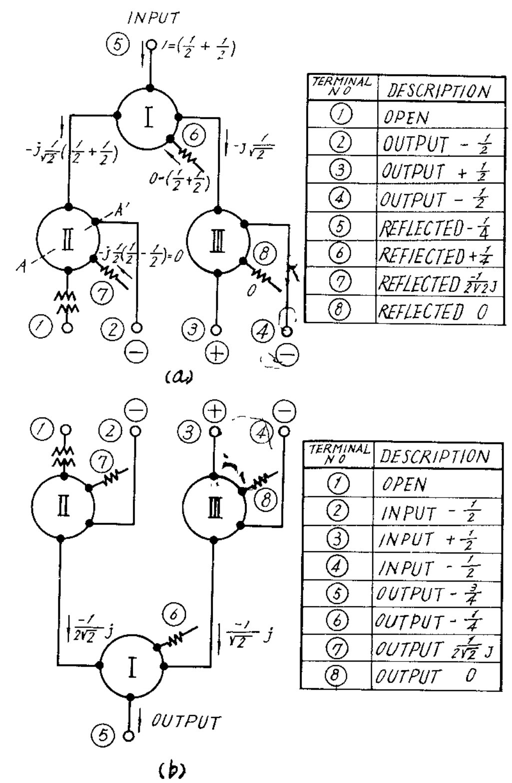


Fig. 5—Parallel push-pull branching and combining circuit using rat-race circuits. (a) "Rat-race" branching circuit. (b) "Rat-race" combining circuit.

fairly well with the theoretical values obtained by Albanese and Peyser.[4] Since rat-race circuits are used in two-stage cascade connection, the frequency bandwidth becomes narrower. Note that the abscissa is normalized by the center frequency.

B. Performance of this Parallel Push-Pull Hybrid Circuit

Consider the performance of a branching and combining circuit using the parallel push-pull hybrid circuits as shown in Fig. 2.

1) Input Branching Circuit: The branching and combining performance is analyzed for a case in which connection of terminal ③ only is opened. Then the input signal 1 may be regraded as equivalent to $+\frac{1}{2}$, $+\frac{1}{2}$ applied to terminal ① and $+\frac{1}{2}$, $-\frac{1}{2}$ applied to terminal ④. Thus the cophase components behave in the same manner as if the line were short-circuited at both points A and B, and these components appear at terminals ⑤ and ⑥ just as in the normal case.

For the antiphase signal component traveling to the right-hand side, the voltage distribution will become identical with that which would be obtained if the line were opened at point A.

2) Output Combining Circuit: An analysis is made of a case in which terminal ③ is opened and the output signals mentioned in Section V-B,1), are applied to terminals ②, ⑤, and ⑥ as shown in Fig. 6(b).

For signal waves from terminals ⑤ and ⑥, the hybrid circuit behaves in the same manner as if the line were short-circuited at points A and B, with the result that signals $-\frac{1}{2}$, $-\frac{1}{2}$ appear respectively at terminals ① and ④.

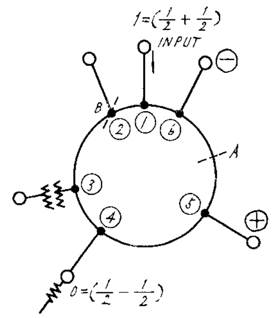
For a signal from terminal ②, the circuit behaves in the same manner as if the circuit were opened at point A and, hence, the equivalent VSWR at terminal ② becomes 2 for the hybrid circuit of Fig. 2.

The combined output power falls off only to 90 per cent of the normal power, which is larger by 2 db than that for the case of using the rat-race circuits. The VSWR of the output combining circuit is lowered to 1.4 as compared with $1.6\dot{6}$ mentioned previously. These relations are the same no matter which terminal connection is opened.

3) Input and Output Impedance Characteristics: For matching and phase adjustment of each signal, it is desirable that the frequency bandwidth for the impedance characteristics of both the branching and combining circuits be quite broad.

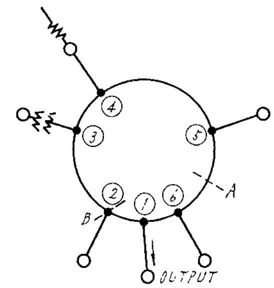
Referring to Fig. 7, Ⓐ shows the input impedance characteristics curve plotted by actual measurements looking into the rat-race circuit from terminal ①, while Ⓑ shows the corresponding curve looking into the parallel push-pull hybrid circuit from terminal ① (shown in Fig. 2).

[4] V. J. Albanese and W. P. Peyser, "An analysis of a broad-band coaxial hybrid ring," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 369–373; October, 1958.



Terminal No	Description
①	REFLECTED $-\frac{1}{6}$
②	OUTPUT $-\frac{2}{3}j$
③	OPEN
④	REFLECTED $-\frac{1}{6}$
⑤	OUTPUT $+\frac{1}{2}j$
⑥	OUTPUT $-\frac{1}{2}j$

(a)



Terminal No	Description
①	OUTPUT $-\frac{17}{18}$
②	INPUT $-\frac{2}{3}j$ REFLECTED [illegible]
③	OPEN
④	OUTPUT $-\frac{1}{18}$
⑤	INPUT $+\frac{1}{2}j$
⑥	INPUT $-\frac{1}{2}j$

(b)

Fig. 6—Branching and combining circuits using parallel push-pull hybrid circuits. (a) Branching circuit. (b) Combining circuit.

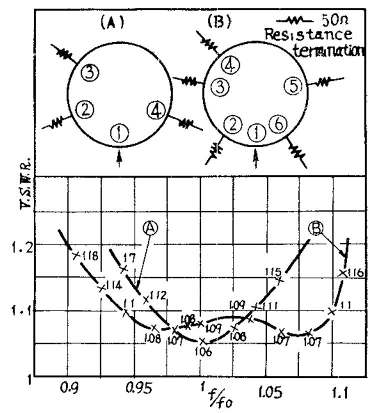


Fig. 7—Input impedance characteristics.

C. Output Stability Characteristics

Provided that each terminal impedance is properly maintained, the combined output is the same no matter which combining system is used, and the output can be computed from the result mentioned in Section III.

Fig. 8 illustrates some typical examples showing variations of the combined output obtained by computation. Fig. 8 (a) shows a case in which the phases of signals to be combined deviate respectively θ_1, θ_2, θ_3 and θ_4 from the normal phases, and θ_1 and θ_2, θ_3 and θ_4 are paired in combining operation.

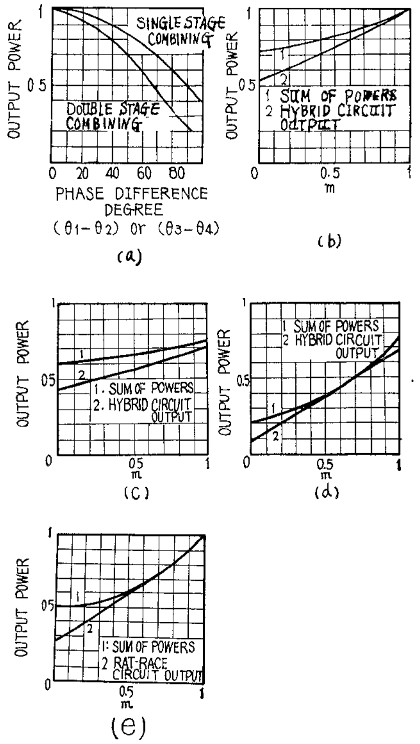


Fig. 8—Some typical examples showing in combined output under various circumstances (calculated values). (a) Decrease in output vs phase difference. (b) When power of one tube only is decreased. (c) When power of one tube falls off to one-half, another less than one-half, the remaining tubes being unchanged. (d) When powers of a pair of tubes are decreased, while powers of the other pair fall off to one-half of the normal. (e) When power of one tube alone varies in push-pull amplification in using rat-race circuits. *Note:* For m, see Section IV-A. Output power is normalized to unity with reference to normal output.

In Fig. 8(b)–(e), typical examples of combining unbalanced signals, the unbalance factor m is taken as the abscissa. Since the abscissa will have to be represented in terms of m^2 in converting voltage to power, the combined output becomes approximately equal to the arithmetic sum of individual tube output powers. The ordinate is normalized to unity with reference to the normal power.

VI. Conclusion

The new parallel push-pull circuit was applied to a 1700-Mc band over-the-horizon communication system as a power amplifier circuit. The circuit construction as shown in Fig. 2 was adopted for both the input branching and output combining circuits, so as to provide 4-tube parallel push-pull operation. When type LD-497 disk-sealed tubes of NEC make were used, a transmitting output of approximately 100 w was obtained. When type LD-531 disk-sealed tubes were used, a transmitting output of as much as 400 w was developed.

Branching or combining of multipair signals is an important part of communication techniques in the diversity systems, in particular, in over-the-horizon communictions. It is conceivable that this circuit can find extension to more than two pairs of signals, and the author hopes to publish a detailed description of such an extension.

In concluding this report, the author also wishes to acknowledge the instructions and encouragement given by Dr. M. Morita and the assistance given by M. Takagi, both of Nippon Electric Company.